ABSTRACT

A case history of a static evaluation of some shallow caves in the karstic geologic region of East Tennessee is presented. The evaluation consists of three parts: (1) characterization of the bedrock voids and openings, (2) rock characterization, and (3) finite element (FE) stress analysis. The bedrock void and rock characterizations are performed to determine the representative bedrock void geometry and to estimate rock mass properties taking into account the frequency and orientation of fractures, the condition of the rock along fractures and the groundwater conditions. On the basis of these conditions a failure criterion is established. Finite element analysis was used to estimate the stresses in the bedrock surrounding the bedrock voids, given the weight of the rock itself as well as an increase in surcharge loading due to a landfill constructed above the karst bedrock. Through an iterative FE analysis process, stress-redistribution was modulus within the tension zones to account for softening around the bedrock void, arching of stresses was modeled to determine the relationship between the location of the softened zone and the upper rock surface. Instability within the bedrock voids was defined by the intersection of the softened zone and upper rock surface.

INTRODUCTION

In the late 1970’s an industrial manufacturing facility with its own coal-fired power generation operations undertook a plan to manage their residual waste (fly ash, bottom ash and other inert incineration by-products) by dry landfilling. The industrial facility acquired property in close proximity to the plant site and began to dispose of the ash waste on the property. At the time of inception, the non-hazardous wastes were managed using common civil engineering waste-handling and disposal protocols. To comply with the promulgated solid waste permit, the industrial facility applied for and received a solid waste permit in the mid-1980’s for the existing landfill. Through the intervening years, other approved, non-hazardous wastes were added to the waste stream. In addition, the regulatory climate in the United States developed to where substantial regulation came into effect. 
play long before the potential disposal volume of the property was exhausted. Accordingly, the industrial facility chose to embark on a permitting program to construct and operate a lateral expansion of the existing landfill that would provide an increase in permitted airspace and meet the facility’s long-term waste disposal needs.

While the property had been acquired in a period when landfill use was perhaps the optimal use of the site, the property did have several physical characteristics that, in today’s regulatory environment, presented a concern to the regulatory community. Specifically, the property is underlain by carbonate rock units of the Knox Group, a series of units notorious for developing karst features. Indeed the property does present the classic karst landform, characterized by coalescing and overlapping closed depressions, intense internal drainage and an underlying network of interconnected joints, cavities and caves in which conduit groundwater flow occurs.

As expected, the regulatory authority was concerned about constructing and operating a lateral expansion given a broad spectrum of potential problems unique to karst areas. These concerns included collapse of soil cavities due to soil loss into rock cavities; rapid movement of potential contaminants through the subsurface; ability to monitor groundwater quality; and finally, the potential for collapse of known openings and unknown caves under their own weight and under the superimposed weight of the landfill components. The first three concerns were addressed by a series of tests and demonstrations which included: (1) tests to show a virtual absence of soil overburden within the lateral expansion area and that the surface landform closely reflected the upper surface of the bedrock; (2) exhaustive hydrogeological investigation, that involved dye tracing and characterization of springs, to ensure both the monitorability and knowledge of the direction and rate for groundwater flow for virtually all hydrologic conditions. These concerns were all addressed to the satisfaction of the regulatory agency up to the point of demonstration of bedrock void stability. The initial stability analysis used finite element (FE) models to estimate the stress conditions based on the Mohr Coulomb failure criteria. The Mohr Coulomb failure criteria, including the tensile limit, had been determined based on lab testing of intact rock specimens. Subsurface explorations performed subsequent to the initial analysis indicated that factors, such as rock quality and jointing, may control cave stability. To evaluate the jointed rock composing the rock surrounding the caves several analytical approaches were considered including beam-column analysis (Tharp, 1995) and empirical analogies with mining conditions (Kesseru, 1997). After consideration of the available alternatives, the Hoek-Brown failure criterion was implemented in the FE analysis. The implementation of the Hoek-Brown failure criterion and the methodology for evaluating the static stability of shallow caves is presented by Siegel and Mauldon (in preparation).

As a result the authors developed and present an approach for evaluating the stability of natural underground openings in jointed rock which combines a rock mass classification system, an empirical strength criterion, and stress analysis using the FEM.
TECHNICAL APPROACH

The approach used for stability evaluation of the natural caves at the studied site in rock consisted of three parts: (1) characterization of the openings, (2) rock characterization, and (3) finite element stress analysis. A description of the three elements follows.

**Bedrock Void Characterization.** Bedrock void characteristics were determined by entering and mapping all openings within the landfill expansion area that were physically accessible. Mapping was performed by a certified speleographer using standard methods including tape measure, Suunto compass and altimeter.

In addition to the cave mapping, surface joint surveys and conventional drilling to recover high quality rock core samples were a part of the bedrock characterization. Cave maps containing details on geometry and condition were used in development of representative FE void models. A total of four representative bedrock void geometries were selected based field data, as shown on Figure 1. Where a given geometry occurred more than once, analysis of the critical geometry (i.e. largest opening or most shallow depth) was performed.

**Rock Characterization.** To characterize the rock conditions, a detailed examination of rock cores and comprehensive strength testing of rock cores was made. These examinations included rock cores drilled throughout the landfill expansion area and one specific core from a Test Boring (designated PAC-7) drilled through the roof of an existing cave based on cave mapping data. The size designation of the rock cores were NQ (47.6 mm) with the exception of PAC-7 that was HQ (63.5 mm). On the basis of our examinations and test results, the bedrock was classified according to the RMR System (Bienawski, 1989) based on the following six parameters: (1) intact rock strength, (2) rock quality, (3) joint spacings, (4) joint conditions, (5) groundwater conditions, and (6) joint orientations. Details of these parameters follow.

**Intact Rock Strength.** The intact strength of the bedrock was estimated by uniaxial compressive tests (ASTM D3148). For this test, select rock core samples were cut to a length-to-diameter ratio of approximately two, and their ends were machined to an acceptable flatness. To allow strain measurements during the application of the compressive load, strain gages were affixed to rock core samples. Incremental compressive loads were applied along the axis of the rock core until failure (i.e., fracture). The uniaxial compressive strength ($q_u$) averaged 82 MPa (11,900 psi). The test results were also used to determine the unit weight ($\gamma$), modulus of deformation ($E$) and Poisson’s ratio ($\nu$) of intact bedrock. The average results are presented in Table 2.

**Rock Quality Designation (RQD).** Rock Quality Designation (RQD) is defined as the total length of pieces of sound rock core with length greater than four inches divided by the length of the core run (Deere, 1968). The average RQD for all of
the core samples, weighted by run length, is 83%. If the upper most core run is not considered (since the upper rock is typically irregular), the average RQD becomes 92%. According to Deere and Deere (1989), an RQD above 90% classifies the bedrock as “excellent” with respect to stability of underground openings. Although the average RQD is representative of the general bedrock conditions, cave roof conditions may be better representative by the conditions at Test Boring PAC-7. Test Boring PAC-7 was drilled through the longest spanning cave roof (based on observed field cave mapping) and the resulting average RQD (not considering the upper-most core run) is approximately 36%. For our cave stability analysis, we conservatively use an RQD of 36 % as our lower bound RQD value.

**Joint Spacings.** For purposes of the stability evaluation, joint spacings were quantified by measuring the distance between similar joint types (e.g., near-vertical joints, along-bedding joints, and cross-bedding joints) in the rock cores supplemented by observations of rock outcrops. The majority of the discontinuity spacings were relatively evenly distributed from 2.0 m to 0.05 m (6.6 ft and 0.2 ft).

**Joint Conditions.** The condition of the joints includes roughness, weathering and separation. Observation of the rock cores indicated that the majority (approximately 53%) of the joints were healed with calcite filling, and that the remaining joints ranged in roughness, degree of weathering and separation. An indication of the high strength of the healed discontinuities was that, in many cases, mechanical breaks in the rock core (based on observations of the uniaxial compressive tests) occurred through continuous rock rather than along nearby healed fractures. Comparison between Boring PAC-7 and the results of all of the borings indicated that PAC-7 has a significantly higher percentage of highly weathered discontinuities.

**Groundwater.** Groundwater study of the site indicated that the groundwater table occurs beneath the depths of the cave zones. Therefore, groundwater flow within the saturated aquifer zone occurs at depths well below the cave zones which are generally observed closer to the existing ground surface. Furthermore, stormwater control measures will be implemented during construction of the proposed landfill. For these reasons, the cave conditions are expected to be moist, but not inundated, except in unusual (some storm events) circumstances.

**Joint Orientations.** The RMR classification includes an adjustment for the orientation of the joints. This adjustment considers steeper dip angles as less favorable. It also considers an orientation of the opening parallel to strike as less favorable. The latter adjustment is more applicable to man-made openings (e.g., tunnels) in which the orientation of the opening is less variable than naturally occurring caves. Because the cave passages at the site represent a range in orientations, we conservatively assigned a rating of *fair* in determination of the RMR.
**Rock Mass Rating.** The RMR system (Bieniawski, 1987), through the use of the aforementioned six parameters, represents the qualitative engineering classification of the rock mass, taken as a whole. For the studied site and the characterize bedrock voids, the RMR values shown in Table 3 are applicable. It is useful to consider the results for Boring PAC-7 separately (as a conservative estimation of rock mass strength) as well as including PAC-7 in the overall rating. For all of the borings, an RMR of 65 was assigned. For PAC-7, an RMR of 50 was assigned.

**Hoek-Brown Failure Criterion.** While the RMR is a very comprehensive analysis of rock mass characteristics and condition, the output is still largely qualitative. A quantitative link to the strength of the rock mass was needed. For that the authors turned to the Hoek-Brown failure criterion (Hoek and Brown, 1980a; Hoek and Brown, 1980b; Hoek and Brown, 1997). The Hoek-Brown failure criterion is an empirical approximation of the strength of jointed rock masses, and is defined, in terms of the major and minor principal stresses, allowing development of a failure envelope. The failure envelope is developed through the following equation:

\[
\sigma_1 = \sigma_3 + (mq_u\sigma_3 + sq_u^2)^{1/2}
\]  

where \(\sigma_1\) and \(\sigma_3\) are the major and minor principal stresses, respectively, \(q_u\) is the uniaxial compressive strength of the intact rock, and \(m\) and \(s\) are empirical constants which depend on the rock type and condition of the rock mass.

As suggested by Hoek and Brown (1980), the dimensionless empirical strength constants \(m\) and \(s\) may be estimated using the following correlations with RMR:

\[
m = m_i \exp \left( \frac{\text{RMR} - 100}{28} \right)
\]  

and

\[
s = \exp \left( \frac{\text{RMR} - 100}{9} \right)
\]

where the dimensionless constant \(m_i\) is determined by triaxial testing. From statistical analysis of published triaxial strength data, Brady and Brown (1993) presented the values of \(m_i\) as a function of rock type shown in Table 3. On the basis of the RMR values, the carbonate rock type and equations (2) and (3), the authors determined the variables for development of the failure envelope [equation (1)] to be as shown in Table 4. With this quantitative measure of rock strength, the authors were prepared to complete the study with the final step, Finite Element Modeling.
**Finite Element Modeling**

The application of FE analysis to evaluate rock stresses and stability is not new to the engineering field. Previous uses of FE analysis include modeling rock as a ‘no tension’ material (Zienkiewicz et al., 1968), modeling joint behavior (Goodman et al., 1968), and stress analysis for underground excavations (Kovari, 1977; Wittke, 1977; Louhenapessy, 1999; Uddin, 1999). The authors’ use of FE analysis is summarized by the following steps:

1. Prepare 2-dimensional mesh (or 3-dimension mesh, if necessary) to represent the four representative bedrock voids determined from mapping (see Figure 1).
2. Estimate initial stresses by performing gravity turn-on (elastic) analysis. This requires appropriate values of deformation modulus (E), Poisson’s ratio (ν), unit weight (γ), and horizontal pressure coefficient (K) for the rock mass.
3. Estimate *in situ* stresses by removing the cave elements and adding external loads (e.g., surcharge from an overlying structure).
4. Assign a reduced modulus of deformation to elements exhibiting a state of stress which exceeds the failure criterion. The reduced modulus represents softening by promoting arching of stresses around the bedrock void to nearby non-yielded elements.
5. Repeat steps 3 and 4 until either (a) solution of the mesh converges to a condition where the stress state of all the elements is within the strength criterion limits, or (b) the reduced modulus zone extends to the top of the rock surface which is interpreted as possible bedrock void instability.

The authors acknowledge that the actual *in situ* rock stresses, particularly at greater depths, are influenced by several conditions (e.g., fault activity, jointing and folding) not considered in this FE analysis. Considering that cave stability is typically a concern for relatively shallow depths [less than 50 m (164 ft)], this (elastic) stress analysis is believed to provide acceptable results. With respect to selection of an appropriate “softened” modulus, it is the relative change from the initial (non-yielded) modulus which is most important to the stress analysis. Consequently, the change in moduli may be initially determined using published correlations with rock type and condition (e.g., Goodman, 1989); however, the moduli values should be adjusted to assure development of arching.

The need to iterate the elastic analysis may be eliminated by selecting a FE program with a softening model; however, use of a relatively simple elastic model has advantages. FE software for elastic analysis is readily available and robust. Also, the elastic model allows easy comparison to published solutions (Poulos and Davis, 1974).

The initial stresses were determined based on the unit weight of the bedrock and a horizontal pressure coefficient (K) of 2.0 which is consistent with field
measurements (Hoek and Brown, 1980a). Conservatively, the authors proposed using a maximum height of the landfill expansion as the surcharge applied to the upper boundary of the FE models. On the basis of a unit weight of 14.1 kN/m³ (90 pcf) and a maximum height of 53.4 m (175 ft), the applied vertical pressure is 754.1 kPa (15,750 psf). The resulting stresses were checked for tension, and where present, a reduced modulus of 1.4 x 10⁴ MPa (2.1 x 10⁶ psi) was assigned to the elements in the tension zone. The reduced modulus values were assigned using correlations with RMR (Bienawski, 1989). The results of using a reduced modulus were that the stresses tended to arch around the tension zone and that much smaller tension forces developed. To check stability of the void roof, the stresses from the FE analysis were compared to the Hoek-Brown failure criterion.

DISCUSSION OF FINDINGS

The stress conditions for the different cave models were plotted in terms of major and minor principal stresses at each node. One representative plot is shown on Figure 2 which indicates that the stress conditions at all node locations within the walls and roofs of the cave models are below and to the right of (i.e. the stresses are less than) the failure envelope defining even the more conservative RMR=50 failure criterion. Furthermore, all of the nodal stresses are well below the less conservative RMR=65 failure criterion. Plots were developed for each of the cave configurations; however, space limitations would not allow all stress plots to be presented herein. Plots for each cave model all showed similar grouping of data points (i.e., nodal stress below the failure envelope). The iterative process of reducing the modulus for the tension zone resulted in a limited softened zone around the cave opening; however, the softened zone did not extend to the upper rock surface in any of the cave models. In conclusion, results of the stability evaluation indicate that potential for collapse of bedrock voids is minimal due either to their own geometry (including jointing) and weight, or due to the surcharge from the proposed landfill components. Thus the results demonstrated that the site exhibits stable bedrock voids for either the existing or fully loaded condition.

CONCLUSIONS

The authors used an approach for stability evaluation of natural underground openings (i.e., caves) in jointed rock. The approach consists of three parts: (1) characterization of the bedrock voids, (2) rock characterization, and (3) finite element stress analysis. The rock characterization is performed to determine the representative geometry and to estimate rock mass properties including the failure criterion. Finite element analysis with an elastic constitutive model is used to estimate the rock stresses. Through an iterative process, stress-redistribution is allowed to determine if stable stress conditions can be achieved.

The advantages of this approach are: (1) the rock behavior (i.e., strength and stiffness) is determined based on readily measurable parameters, (2) the flexibility of FE analysis allows consideration of irregular shaped openings, and (3)
elastic models are relatively simple and readily available in most FE software. While this approach has advantages in comparison to previous approaches to natural openings, there are limitations, mainly related to its level of complexity. These limitations include: (1) no consideration of the intermediate principal stresses, (2) no attempt to model actual crack propagation, and (3) the iterative process using a softened modulus. Some or all of these disadvantages may be overcome by selecting a more sophisticated constitutive model; however, greater modeling sophistication usually requires more involved testing.

ACKNOWLEDGEMENTS

The authors wish to acknowledge the great contributions of Matthew Mauldon to our comprehension of the overlapping methodologies used herein.

REFERENCES


TABLE 1. Hoek-Brown Strength Constant $m_i$ (Brady and Brown, 1993)

<table>
<thead>
<tr>
<th>Type of Rock</th>
<th>$m_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbonate rocks with well developed crystal cleavage (e.g., dolomite,</td>
<td>7</td>
</tr>
<tr>
<td>limestone and marble)</td>
<td></td>
</tr>
<tr>
<td>Lithified argillaceous rocks (e.g., mudstone, siltstone, shale and slate)</td>
<td>10</td>
</tr>
<tr>
<td>Arenaceous rocks with strong crystals and poorly developed crystal cleavage</td>
<td>15</td>
</tr>
<tr>
<td>(e.g., sandstone and quartzite)</td>
<td></td>
</tr>
<tr>
<td>Fine-grained polyminerallc igneous crystalline rocks (e.g., andesite, diabase</td>
<td>17</td>
</tr>
<tr>
<td>and rhyolite)</td>
<td></td>
</tr>
<tr>
<td>Coarse-grained polyminerallc igneous and metamorphic rocks (e.g.,</td>
<td>15</td>
</tr>
<tr>
<td>amphibolite, gabbro, gneiss, granite and quartz-diorite)</td>
<td></td>
</tr>
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</table>

TABLE 2. FE Analysis Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Unit Weight ($\gamma$)</td>
<td>26.7 kN/m$^3$ (170 pcf)</td>
</tr>
<tr>
<td>Horizontal Stress Coefficient ($K$)</td>
<td>2.0</td>
</tr>
<tr>
<td>Initial Modulus of Deformation ($E$)</td>
<td>$5.5 \times 10^4$ MPa (8 x $10^6$ psi)</td>
</tr>
<tr>
<td>Softened Modulus of Deformation ($E_s$)</td>
<td>$1.4 \times 10^4$ MPa (2.1 x $10^6$ psi)</td>
</tr>
<tr>
<td>Poisson’s Ratio ($\nu$)</td>
<td>.31</td>
</tr>
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</table>
TABLE 3. RMR Values

<table>
<thead>
<tr>
<th>RMR Parameter</th>
<th>All Borings (1)</th>
<th>PAC-7 (2)</th>
</tr>
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<tbody>
<tr>
<td>Strength of Intact Rock</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>RQD</td>
<td>18</td>
<td>7</td>
</tr>
<tr>
<td>Spacing of Discontinuities</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Condition of Discontinuities</td>
<td>20</td>
<td>16</td>
</tr>
<tr>
<td>Groundwater</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Orientation of Discontinuities</td>
<td>-5</td>
<td>-5</td>
</tr>
<tr>
<td><strong>RMR</strong></td>
<td><strong>65</strong></td>
<td><strong>50</strong></td>
</tr>
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TABLE 4. Hoek-Brown Strength Criterion Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>All Borings (1)</th>
<th>PAC-7 (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_u$</td>
<td>82 MPa (11,900 psi)</td>
<td>Not Tested</td>
</tr>
<tr>
<td>$m_j$</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>$m$</td>
<td>2.01</td>
<td>1.17</td>
</tr>
<tr>
<td>$s$</td>
<td>.02</td>
<td>.004</td>
</tr>
</tbody>
</table>
FIG. 1. Cave Models (a) Long Narrow Inclined Cave; (b) Long Arched-Roof Cave; (c) Long Narrow Vertical Cave; (d) Ellipsoidal Arched-Roof Cave
FIG. 2. Estimated Stress Conditions for Model 1 - Long Narrow Inclined Cave