Probabilistic Calibration of Resistance Factors for Slope Stability

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ABSTRACT: Significant research has been performed in recent years to develop load and resistance factor design (LRFD) methods for geotechnical applications. However, one lacking aspect of this work is application to design of earth slopes, including consideration of the overall stability of earth retaining structures. A procedure for implementing LFRD for slope stability evaluations is proposed in this paper, using "probabilistically calibrated" resistance factors established to achieve a range of target probabilities of failure. Results of probabilistic calibrations demonstrate that LRFD methods can be used for overall stability evaluations without unnecessary complications. Appropriate resistance factors were found to be insensitive to slope geometry but dependent upon the relative magnitude of soil strength parameters. Fortunately this dependence can be practically addressed using Janbu's dimensionless stability parameter, $\lambda_{c\phi}$, which in turn allows LRFD techniques to be feasibly implemented for slope stability applications.

INTRODUCTION

The American Association of State Highway and Transportation Officials (AASHTO) is currently working to progressively convert from traditional Allowable Stress Design (ASD) methods to Load and Resistance Factor Design (LRFD) methods. This conversion began with the adoption of the *LRFD Highway Bridge Design Specifications* in 1994 and has progressed since then to incorporate more applications, including many geotechnical applications such as shallow and deep foundations, earth retaining structures, and culverts. The principal motivation behind transitioning to LRFD is to produce consistent levels of reliability (or safety) across a broad range of design cases, regardless of the level of uncertainty associated with loads and resistances for a particular case. The fundamental promise of transitioning to LRFD is that substantial cost savings can be realized by applying appropriate conservatism (and thus funding) where needed but not where not needed. This promise cannot be realistically achieved using existing design approaches that lump conservatism into a single factor of safety. LRFD also offers the potential for continuous improvement of design methods over time without requiring wholesale changes to formalized design procedures.

While progress has been made developing LRFD methods for geotechnical applications, little work has been performed for design of earth slopes, including consideration of the overall stability of earth retaining structures. This paper proposes a procedure for implementing LFRD for slope stability evaluations. A procedure for probabilistic calibration of load and resistance factors is then presented along with results of calibrations performed to achieve various target reliabilities

GENERAL PROCEDURE FOR STABILITY EVALUATIONS USING LRFD

The procedure for implementing slope stability evaluations using the LRFD approach is similar to the conventional approach used for ASD evaluations. The general LRFD procedure involves the following steps:

1. Establish site geometry using available geologic information, boring logs, site surveys and plans, and other available information;
2. Estimate parameters for each stratum within the slope using laboratory test results, empirical correlations, and other available information;
3. Estimate pore pressure conditions based on available historical records, analyses, and judgment (required only for effective stress analyses);
4. Evaluate the factor of safety for the conditions established using common slope stability analysis methods with *factored parameters* as input; and
5. Compare the computed factor of safety to the limit factor of safety (= 1.0):
    a. If the computed factor of safety is approximately equal to 1.0, the design is considered acceptable.
    b. If the factor of safety is significantly greater than 1.0, changes to reduce the computed factor of safety should be considered if significant cost savings can be realized.
    c. If the factor of safety is less than 1.0, the designer must consider alternative measures to increase the factor of safety and repeat the procedure until a factor of safety approximately equal to 1.0 is achieved.

The primary changes from current practice involve Steps 4 and 5. The first difference occurs in Step 4, where factored parameters are used as input for slope stability analyses in the LRFD procedure whereas unfactored parameters are used in current practice. The second difference occurs in Step 5, where the computed factor of safety is compared to a limit value (= 1.0) indicating stability or instability rather than being compared to a target value of the factor of safety (typically between 1.3 and 1.5). In this respect, the LRFD procedure is more straightforward than current procedures in that the analysis target or limit is consistent for all stability cases. The result of these differences is that uncertainties in the analyses are addressed by factoring input parameters for LRFD procedures whereas uncertainty is accounted for using a single factor of safety in ASD. By factoring individual input parameters, it is possible to more appropriately apply conservatism to the individual parameters involved in the analysis, and therefore to effect more consistent levels of safety across a broad range of cases. Both load and resistance factors in LRFD and factors of safety in ASD are intended to account for uncertainties involved in the respective analyses. They are simply different methods to account for these uncertainties.

Five common parameters are typically used as input for slope stability analyses[1]. These parameters include the soil (total) unit weight, γ, undrained shear strength, s_u, Mohr-Coulomb shear strength parameters, c and ϕ (or c' and ϕ' in the case of effective stress analyses), and the pore water pressure, u. For the purposes of this paper, pore water pressures will be considered as deterministic values that are not factored. The same is assumed of site stratigraphy and slope geometry. The remaining input parameters are consider probabilistic and are factored for LRFD analyses. Factored values are established by first estimating appropriate nominal, or mean values for the required parameters following conventional approaches (laboratory tests, field tests, empirical correlations, etc). These nominal values are then "factored" using calibrated "load factors" or "resistance factors" to produce factored parameters that are subsequently used as input for slope stability analyses.

It is generally logical to adopt soil unit weight as being a "load" while soil shear strength or shear strength parameters are considered to be "resistance". Thus, factored values of soil unit weight are computed as

$$\gamma^* = \chi \cdot \gamma \tag{1}$$

where γ^* is the factored unit weight to be used in stability analyses for LRFD, χ is the load factor and γ is the unfactored nominal unit weight. Value for load factors will generally be greater than or equal to 1.0, although this is not necessarily required.

Similarly, factored values for undrained shear strength are computed as

$$s_u^* = \psi \cdot s_u \tag{2}$$

where s_u^* is the factored undrained shear strength, ψ is the resistance factor for undrained strength, and s_u is the unfactored, nominal undrained shear strength. For cases where the shear strength is represented using a Mohr-Coulomb failure envelope and shear strength parameters c and ϕ for total stress analyses or $\bar{c}$ and $\bar{\phi}$ for effective stress analyses, factored shear strengths can be effected by applying the resistance factors to the respective parameters independently. Thus, factored shear strength can be achieved by inputting factored parameters c^* and ϕ^* computed as

$$c^* = \psi \cdot c\,; \quad \tan\phi^* = \psi \cdot \tan\phi \tag{3}$$

[1] Other parameters such as reinforcement loads, surcharge loads, etc. may also be required in some cases but are not considered here

where c and ϕ are the unfactored shear strength parameters defined in terms of total stresses. Calculations can be similarly made for effective stress strength parameters.

CALIBRATION OF LOAD AND RESISTANCE FACTORS

The principal objective of calibrating load and resistance factors for LRFD is to produce factors that, when used in combination, will produce designs that achieve some target level of reliability. One key aspect of LRFD is that both load and resistance factors are "coupled" to produce appropriate reliability. Thus, the process of calibration is in general non-unique. One could conceptually provide the entire margin of safety exclusively through load factors, exclusively though resistance factors, or through some combination of factors. The preferred combination of factors is the one that applies margins of safety that are consistent with knowledge of the respective loads and resistances. However, this is complicated by the need to limit the complexity of design to a reasonable level and to maintain some degree of consistency across design for different applications. As a result, final implementations of LRFD generally represent compromise of competing demands.

Current implementations of LRFD for applications involving the stability of spread footings and classical retaining walls, both of which require evaluation of overall slope stability, have recommended that stability be evaluated under what is referred to as the Service I limit state (FHWA, 2001). In this limit state, the load factors applied to the unit weight of the soil is generally taken to be equal to 1.0. This approach is somewhat crude in the sense that some of the uncertainty in these designs can certainly be attributed to uncertainty in the unit weight of the soil. However, as a practical matter, uncertainty in unit weight is generally very small when compared to uncertainty in other parameters so adopting a χ=1.0 is a reasonable simplification. To be consistent with these existing design specifications, and as a reasonable first approximation, calibrations for resistance factors have been performed utilizing a load factor of χ=1.0 for the unit weight of the soil. Alternative load factors may be developed at a later time to better account for potential uncertainty and variability in soil unit weight, but it should be noted that this will also require re-calibration of resistance factors since the two types of factors are coupled.

Calibration Procedure

The procedure for probabilistic calibration of load and resistance factors to produce a target reliability includes the following steps:

1. Select a target probability of failure (or reliability, reliability index, etc.);
2. Establish the range of possible conditions for which designs may be performed including the range of slope angles, slope heights, soil strengths, foundation conditions, etc;
3. For selected cases representative of this range of conditions:

a. Determine several combinations of means and *COV* (or standard deviations) of the input parameters that produce the target probability of failure. This is most effectively done by selecting values of *COV* that span the range of expected *COV*s for the respective parameters and then varying the mean values to produce the target probability of failure.
b. Establish the deterministic values of the input parameters that produce a factor of safety of unity (1.0).
c. For each combination of mean and standard deviation from step 3a, compute the appropriate resistance factor for each parameter as

$$\psi(COV) = \frac{x_{\det}}{\mu_x(COV)} \quad (4)$$

where x_{det} is the deterministic value of a particular input parameter producing a factor of safety of 1.0 and μ_x is the mean value of the same parameter producing the target value of the probability of failure. Since μ_x is dependent on the *COV* of the parameter, the computed resistance factor is also a function of the *COV*.

This procedure is straightforward but poses two practical challenges. The first is that Step 3a requires a laborious trial and error process to define load or resistance factors. The second challenge is addressed in the following section.

Selection of Slope Conditions for Calibration

Perhaps the most difficult challenge for calibration of load and resistance factors for slope stability applications is establishing an appropriate range of possible conditions to be evaluated to ensure that the developed factors are appropriate for the conditions that will be encountered. Even for homogeneous slopes, the stability of a slope is dependant on seven parameters that include (Bishop and Morgenstern, 1960): slope height, *H*; slope angle, β; shear strength parameters, *c* and ϕ; soil unit weight, γ; depth to a "hard stratum", *d*; and pore water pressure, or one or more parameters representing pore water pressures (e.g. r_u). For stratified sites, the number of parameters increases even further making the problem seem almost intractable. However, the problem is made more tractable by considering the following:

1. The stability of a homogeneous slope can be represented as a function of the dimensionless parameter $\lambda_{c\phi}$ defined as

$$\lambda_{c\phi} = \frac{\gamma \cdot H \cdot \tan\phi}{c} \quad (5)$$

the slope angle, β, the pore pressure, and the ratio $c/\gamma H$ (Duncan and Wright, 2005), thus reducing the number of parameters from seven to four.
2. It is the uncertainty in stability, which may be dominated by relatively few input parameters, that dictates appropriate load and resistance factors.

Based on these considerations, the general slope geometry depicted in Fig. 1 was utilized, with appropriate modifications as warranted, as the basis for calibration of load and resistance factors presented subsequently. Modifications to this geometry generally included varying the slope height and the slope angle, as well as considering a case where the slope was formed as a vertical earth retaining structure. The effect of such modifications is described subsequently.

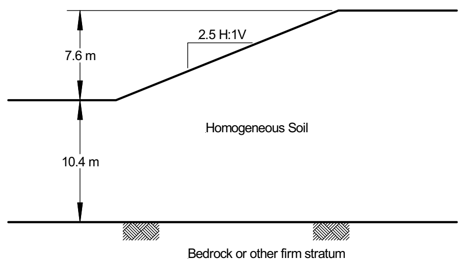

Fig. 1. General slope geometry used for calibration of load and resistance factors.

Calibrations were also performed for a range in values of the parameter $\lambda_{c\phi}$ to account for potential variations in material properties. In general, $\lambda_{c\phi}$ varies from a value of 0 for saturated slopes under undrained conditions (i.e. ϕ=0 condition) to a value of infinity when the cohesion intercept, *c*, is zero. Calibrations were therefore performed for these extreme values of $\lambda_{c\phi}$=0 and $\lambda_{c\phi}=\infty$. For intermediate values, results of preliminary analyses indicated that results for different values of $\lambda_{c\phi}$ could be reasonably represented by performing calibrations for $\lambda_{c\phi}$=10. In all cases, the total unit weight of the soil was taken to be 19.6 kN/m^3 (125 pcf), and except where noted otherwise, the slope height was taken to be 7.6-m (25-ft). Pore water pressures were taken as zero for all calibrations. Additional assumptions for the calibration analyses include that the coefficients of variation for the cohesion intercept and friction angles were assumed to be the same and that the same resistance factors would be applied to both the cohesion intercept and friction angle.

RESULTS OF CALIBRATION ANALYSES

Fig. 2 shows computed resistance factors for the shear strength parameters plotted as a function of the coefficient of variation of these parameters for target probabilities of failure, p_f, of 1 in 10, 1 in 100, and 1 in 1000. These relations show decreasing resistance factors with increasing coefficients of variation of the shear strength parameters and decreasing target probability of failure. Fig. 2 also shows that lower resistance factors are required for a given target probability of failure as $\lambda_{c\phi}$ increases.

SENSITIVITY OF FACTORS TO SLOPE ANGLE AND HEIGHT

Resistance factor relations presented in Fig. 2 were established for the slope geometry presented in Fig. 1. Fig. 3 shows results from additional analyses performed to evaluate sensitivity to other slope geometries. Results are shown for slope inclinations of 1.5:1, 2.5:1, and 3.5:1 and slope heights of 3.8-m (12.5-ft), 7.6-

m (25-ft), and 15.2-m (50-ft). The results shown indicate that computed resistance factors are insensitive to both slope angle and slope height, which suggests they are applicable for a relatively broad range of slope geometries.

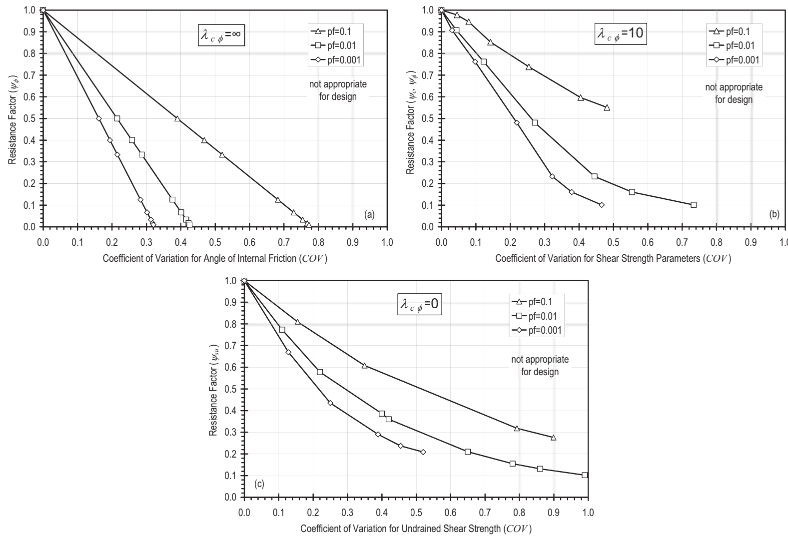

Fig. 2. Resistance factors determined from probabilistic calibrations for: (a) $\lambda_{c\phi} = \infty$, (b) $\lambda_{c\phi} = 10$, and (c) $\lambda_{c\phi} = 0$.

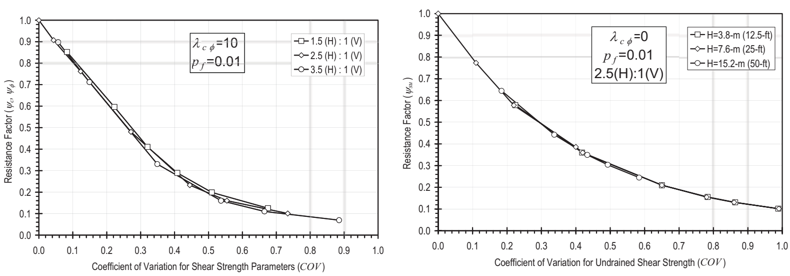

Fig. 3 Comparison of resistance factors for varying slope angle and slope height.

DISCUSSION

Figs. 2 and 3 both indicate that resistance factors necessary to achieve a given probability of failure are quite sensitive to the coefficient of variation of the shear strength parameters. This is logical in that greater resistance factors can be used in cases where soil strength parameters are known with greater confidence. Thus, one can use greater resistance factors for cases where significant effort is taken to complete a high quality site investigation while lesser resistance factors would be required with less rigorous site investigations. Similar, greater resistance factors could be used for relatively uniform sites while lesser resistance factors would be required for more variable sites. This is generally consistent with current ASD practice. The difference is that this relation is quantified with respect to the coefficient of variation in LRFD whereas it is rather nebulous and vague in ASD.

Since the load factor for unit weight was taken to be 1.0, and since the resistance factors for c and ϕ were assumed identical, resistance factors shown in Figs. 2 and 3 are actually the inverse of the conventional factor of safety used in ASD. Thus, a resistance factor of 0.67 in the figures corresponds to an ASD factor of safety of 1.5, a resistance factor of 0.5 to a factor of safety of 2.0, etc. Comparison of the computed resistance factors with commonly used ASD factors of safety (1.3 to 1.5) reveals that the resistance factors appear to be more conservative than current practice when the coefficient of variation for soil strengths is relatively large (say > 20%). However, it is important to realize that additional conservatism is commonly introduced into ASD analyses via selection of conservative strength parameters and other means. Such practice introduces an unquantified bias into conventional ASD calculations. Such bias can also be included in LRFD, but doing so tends to disguise the influence of the respective design parameters and is counter to the general objectives of LRFD.

CONCLUSION

A procedure for evaluation of slope stability using LRFD techniques has been presented in this paper. The principal differences between the LRFD procedure and current practice is that the LRFD procedure utilizes factored input parameters and that the LRFD procedure requires comparison of computed factors of safety with a limit value (=1.0) rather than the target factor of safety used in current practice. Factored input parameters are established using load or resistance factors that are applied to mean or nominal values of input parameters established by conventional means. Results demonstrate that LRFD techniques can be practically applied to slope stability problems. Probabilistically derived resistance factors presented in the paper indicate that resistance factors required to achieve target reliabilities are generally quite sensitive to the variability of the input parameters. However, resistance factors are generally insensitive to slope height and slope inclination, which suggests that they can be used for a broad range of slope stability cases.

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